VIBRATION OF A CLAMPED—CLAMPED BAR UNDER TENSION

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(*Received August* 11, 1965)

ABSTRACT. Dynamics of vibration of a bar clamped at both ends and excited by transverse impact by an inelastic load has been worked out following operational method. We consider in this paper the dynamics of such a bar struck at the mid-point only. Two distinct cases have been worked out: (1) The tension is large compared to stiffness, (2) Stiffness is large compared to the tension. In case (1) series solution is obtained from which the solution for any epoch may be calculated easily. In case (2) a periodic solution is obtained.

INTRODUCTION

The problem of Vibration of a bar Clamped at both ends and held under tension is in effect the same as that of a Stiff—String.

When a string is under tension and also has stiffness, its equation of motion can be obtained by combining the derivatives in connection with the vibration of the string and transverse vibration of a bar. Velocity of the wave remains strictly constant if the wire has no stiffness, but the velocity changes and becomes more and more frequency-dependent if stiffness is considered. The same is true for a bar clamped at both ends, held under tension and vibrating transversely. So we prefer to use the word Stiff-String in place of Clamped—Clamped bar in owr subsequent discussions. The present problem discusses two distinct cases: (1) when the tension is the chief agent in the vibration (2) when the stiffness is the chief agent in the vibration.

In this paper we solve the general problem of vibration of a stiff string excited by the transverse impact by an inelastic load using the powerful operational method in a simmilar way as adopted by Ghosh (1938) in solving the general problem of painoforte string. The only assumption in the present formulation is that the string behaves like a loaded one so long as the load is in contact with it.

EXPLANATION OF THE SYMBOLS USED

l = Length of the string = $a+b$

a = Shorter segment of the string

b = longer segment of the string

t = Variable time

x = Variable, measured along the string, the string being clamped at $x = 0$ and at $x = l$

y = Displacement of any point of the string at any time

y_1 = Displacement of any point, $x < a$

y_2 = Displacement of any point, $x > a$

y_a = Displacement of the struck point $x = a$

A = Area of cross-section of the string

ρ = Density of the material of the string

T = Parmanent tension along the string

c = Velocity of transverse waves along the string $= (T/\rho A)^{\frac{1}{2}}$

E = Young's Modulus of the material of the string

I = Moment of Inertia of the cross-section of the string about the neutral axis.

P = Pressure exerted by the load

m = Mass of the stricking body

$\theta = 2a/c$.

$t_n = t - n\theta$, where $n = 1, 2, 3$ etc.

v_0 = Velocity of Impact

$J = mv_0$

D = Operator $\frac{d}{dt}$.

The equation of motion of the stiff string or, a bar clamped at both ends and held under tension,

$$\rho A \frac{d^2y}{dt^2} = T \frac{d^2y}{dx^2} - EI \frac{d^4y}{dx^4} \tag{1}$$

Equation (1) in the operational notations becomes,

$$EI \frac{d^4y}{dx^4} - T \frac{d^2y}{dx^2} + \rho A D^2 y = 0 \tag{2.0}$$

The general solution (2.0) is,

$$y = A_1 \exp(K_1 x) + A_2 \exp(-K_1 x) + A_3 \exp(K_2 x) + A_4 \exp(-K_2 x) \quad \dots \quad (2.1)$$

where
$$K^2_1 = [T + (T^2 - 4EI\rho AD^2)^{\frac{1}{2}}]/2EI \quad \dots \quad (2.2)$$

$$K^2_2 = [T - (T^2 - 4EI\rho AD^2)^{\frac{1}{2}}]/2EI \quad \dots \quad (2.3)$$

and A_1, A_2, A_3, A_4 are constant to be determined with the help of end-conditions.

Case I : $T^2 > 4EI\rho AD^2$

In this case the root of (2.0) are all real and K_1 and K_2 can be written simply by,

$$K_1 = \pm(T/EI)^{\frac{1}{2}} \quad \dots \quad (2.4)$$

$$K_2 = \pm D/c \quad \dots \quad (2.5)$$

The general solution (2.1) now takes up the form

$$y = A_1 \exp\{(T/EI)^{\frac{1}{2}}x\} + A_2 \exp\{-(T/EI)^{\frac{1}{2}}x\} + A_3 \exp\{(D/c)x\} + A_4 \exp\{-(D/c)x\} \quad \ldots \quad (3)$$

Since the string is clamped at both ends, the terminal conditions are,

$$\text{at} \quad x = 0, \quad y_1 = 0, \quad \frac{dy_1}{dx} = 0 \quad \ldots \quad (3.1)$$

$$x = l, \quad y_2 = 0, \quad \frac{dy_2}{dx} = 0 \quad \ldots \quad (3.2)$$

$$x = a \quad y_1 = y_2 = y_a \quad \ldots \quad (3.3)$$

The hammer strikes at $x = a$; if y_a be the displacement of the struck-point we get from (3), (3.1), (3.2) and (3.3)

$$y_1 = y_a \frac{\cosh (T/EI)^{\frac{1}{2}}x - \cosh (D/c)x}{\cosh (T/EI)^{\frac{1}{2}}a - \cosh (D/c)a}, \quad (0 \leq x \leq a) \quad \ldots \quad (4.0)$$

$$y_2 = y_a \frac{\cosh (T/EI)^{\frac{1}{2}}(l-x) - \cosh (D/c)(l-x)}{\cosh (T/EI)^{\frac{1}{2}}b - \cosh (D/c)b}, \quad (a \leq x \leq l) \quad \ldots \quad (4.1)$$

The string is excited by a transverse impact of impulse J by an inelastic load. The corresponding pressure P exerted by the load is given by,

$$P = -m \frac{d^2y_a}{dt^2} \quad \ldots \quad (5.0)$$

and the subsequent motion of the load during contact is,

$$m \frac{d^2y_a}{dt^2} = T\Delta \left(\frac{dy}{dx} \right)_{x=a} - EI\Delta \left(\frac{d^3y}{dx^3} \right)_{x=a} \quad \ldots \quad (5.1)$$

where $\Delta(dy/dx)_{x=a}$ and $\Delta(d^3y/dx^3)_{x=a}$ denote the changes in the values of (dy/dx) and (d^3y/dx^3) incurred in crossing the point $x = a$.

Now substituting the values of $\Delta(dy/dx)_{x=a}$ and $\Delta(d^3y/dx^3)_{x=a}$ as obtained from (4.0) and (4.1) in equation (5.1) and imposing the boundary conditions we get,

$$mD^2y_a = -[(TD/c) - (EID^3/c^3)] \times$$

$$\left\{ \frac{\sinh (D/c)a}{\cosh (D/c)a - k} + \frac{\sinh (D/c)b}{\cosh (D/c)b - k'} \right\} + DJ \quad \ldots \quad (5.2)$$

Whence we get,

$$y_a = \frac{1}{F(D)} v_0 \quad \ldots \quad (6.0)$$

Where,

$$F(D) = D + \tfrac{1}{2}\{q - (2EI/mc^3)D^2\}\left[\frac{\sinh(D/c)a}{\cosh(D/c)a - k} + \frac{\sinh(D/c)b}{\cosh(D/c)b - k'}\right] \quad \ldots \quad (6.1)$$

and,

$$q \quad 2T \quad \ldots \quad (6.2)$$

$$k = \cosh(T/EI)^{\frac{1}{2}}a \quad \ldots \quad (6.3)$$

$$k' = \cosh(T/EI)^{\frac{1}{2}}b \quad \ldots \quad (6.4)$$

The equation (6.1) is to be used for finding the displacement at any point along the string.

In the absence of stiffness, equation (1) reduces to that of a flexible string but if stiffness is made zero in equation (6.1) we can not expect the behaviour of a flexible string because of difference in end conditions. But it represents the case of a clamped-clamped bar held under high tension [Kar (1956)].

String is finite :

The Hammer strikes at the middle point of the string. Here $a = b$ and we get from (6.1)

$$F(D) = D + \{q - (2EI/mc^3)D^2\}\left[\frac{\sinh(D/c)a}{\cosh(D/c)a - k}\right] \quad \ldots \quad (7.0)$$

On substituting the exponential values for hyperbolic Sines and Cosines in equation (7.0) and writing $D_1 = D + \alpha$, $D_2 = D + \beta$ we get finally.

$$F(D) = \frac{D_1 D_2}{(\alpha + \beta)[1 - 2k\exp\{-(D/c)a\} + \exp\{-2(D/c)a\}]}$$

$$\times\left[1 - \frac{(D-\alpha)(D-\beta)}{D_1 D_2}\left\{1 + \frac{2k(\alpha+\beta)}{(D-\alpha)(D-\beta)}\exp(D/c)a\right\}\right] \quad \ldots \quad (8.0)$$

where, $D_1 D_2 = (D+\alpha)(D+\beta) = D^2 + \dfrac{mc^3}{2EI}D - \dfrac{mqc^3}{2EI}$ $\quad \ldots \quad (9.0)$

and $-\alpha$, $-\beta$ are the roots of. $\quad D^2 + \dfrac{mc^3}{2EI}D - \dfrac{mqc^3}{2EI} = 0 \quad \ldots \quad (10)$

given by,

$$[\alpha, \beta] = \frac{1}{2}\left[\frac{mc^3}{2EI} \pm \left(\frac{m^2c^6}{4E^2I^2} + \frac{2mqc^3}{EI}\right)^{\frac{1}{2}}\right] \quad \ldots \quad (11)$$

Expanding the term under the radical sign, binomially and retaining up to terms containing $(1/mc^3)$, and remembering that, $c = (T/\rho A)^{\frac{1}{2}}$, $q = 2T/mc$, equation (11) reduces to,

$$|\alpha, \beta| = -\left(\frac{T}{\rho A}\right)^{\frac{1}{2}} \left[\frac{2\rho A}{m} + \frac{mT}{2EI\rho A} - \left(\frac{2\rho A}{m}\right)^3 \frac{EI}{T}\right],$$

$$\left(\frac{T}{\rho A}\right)^{\frac{1}{2}} \left[\frac{2\rho A}{m} - \left(\frac{2\rho A}{m}\right)^3 \frac{EI}{T}\right] \quad \ldots \quad (12)$$

Displacement of the struck-point during contact

With the help of equation (8.0) we get from (6.0) the displacement of the string at the struck-point during contact of the load.

$$y_a = \frac{v_0(\alpha+\beta)\left[1 - 2k\exp\left\{-\frac{Da}{c}\right\} + \exp\left\{-\frac{2Da}{c}\right\}\right]}{D_1 D_2}$$

$$\times \left[1 - \frac{(D-\alpha)(D-\beta)}{D_1 D_2}\left\{1 + \frac{2k(\alpha+\beta)}{(D-\alpha)(D-\beta)}\exp\left(\frac{Da}{c}\right)\right\}\right]^{-1} \quad \ldots \quad (13)$$

Expanding the above multmomial expression in power series we get after simplification,

$$y_a = \left[\frac{(\alpha+\beta)}{D_1 D_2} - \left\{\frac{2(\alpha+\beta)^2 D}{D_1^2 D_2^2} - \frac{2(\alpha+\beta)}{D_1 D_2}\right\}\exp(-D\theta) + \left\{\frac{4(\alpha+\beta)^3 D^2}{D_1^3 D_2^3} - \frac{6(\alpha+\beta)^2 D}{D_1^2 D_2^2}\right.\right.$$

$$\left.\left. + \frac{2(\alpha+\beta)}{D_1 D_2}\right\}\exp(-2D\theta) - \ldots\right] v_0 + 2k\left[\left\{\frac{(\alpha+\beta)^2 D}{D_1^2 D_2^2} - \frac{(\alpha+\beta)}{D_1 D_2}\right\}\exp\left(-\tfrac{1}{2}D\theta\right)\right.$$

$$- \left\{\frac{4(\alpha+\beta)^3 D^2}{D_1^3 D_2^3} - \frac{5(\alpha+\beta)^2 D}{D_1^2 D_2^2} + \frac{(\alpha+\beta)}{D_1 D_2}\right\}\exp\left(-\tfrac{3}{2}D\theta\right)$$

$$\left. + \left\{\frac{12(\alpha+\beta)^4 D^3}{D_1^4 D_2^4} - \frac{20(\alpha+\beta)^3 D^2}{D_1^3 D_2^3} + \frac{9(\alpha+\beta)^2 D}{D_1^2 D_2^2} - \frac{(\alpha+\beta)}{D_1 D_2}\right\}\exp\left(-\tfrac{5}{2}D\theta\right) - \right] v_0$$

$$+ 4k^2\left[\left\{\frac{(\alpha+\beta)^3 D^2}{D_1^3 D_2^3} - \frac{(\alpha+\beta)^2 D}{D_1^2 D_2^2}\right\}\exp(-D\theta) - \left\{\frac{6(\alpha+\beta)^4 D^3}{D_1^4 D_2^4} - \frac{8(\alpha+\beta)^3 D^2}{D_1^3 D_2^3}\right.\right.$$

$$\left.\left. + \frac{2(\alpha+\beta)^2 D}{D_1^2 D_2^2}\right\}\exp(-2D\theta) + \ldots\right] v_0 + 8k^3\left[\left\{\frac{(\alpha+\beta)^4 D^3}{D_1^4 D_2^4} - \frac{(\alpha+\beta)^3 D^2}{D_1^3 D_2^3}\right\}\right.$$

$$\exp\ (-\tfrac{3}{2}\ D\theta)\cdot\left\{\frac{8(\alpha+\beta)^5D^1}{D_1^{\,5}D_2^{\,5}}\quad\frac{11(\alpha+\beta)^4D^3}{D_1^{\,4}D_2^{\,4}}+\frac{3(\alpha+\beta)^3D^2}{D_1^{\,3}D_2^{\,3}}\right\}$$

$$\exp\ (-\tfrac{5}{2}D\theta)+\Big]v_0+16k^4\left[\left\{\frac{(\alpha+\beta)^4D^1}{D_1^{\,5}D_2^{\,5}}-\frac{(\alpha+\beta)^4D^3}{D_1^{\,4}D_2^{\,4}}\right\}\exp\ (\ \ 2D\theta)\ \ \ldots\right]v_0\ldots \quad (14)$$

Writing,

$$\frac{(\alpha+\beta)}{D_1D_2}\,v_0=f_1(t)$$

$$\frac{(\alpha+\beta)^2D}{D_1^{\,2}D_2^{\,2}}\,v_0=f_2(t)$$

etc,

$$\frac{(\alpha+\beta)^nD^{n-}}{D_1^{\,n}D_2^{\,n}}\,v_0=f_n(t)$$

and remembering that, $f(t)\exp\ (-nD\theta)=f(t-n\theta)=f(t_n)$, equation (14) reduces to,

$$y_a=[f_1(t)-\{2f_2(t_1)\ \ 2f_1(t_1)\}+\{4f_3(t_2)\ \ 6f_2(t_2)+2f_1(t_2)\}-\ldots]$$

$$+2k\Big[\Big\{f_2\Big(t-\frac{\theta}{2}\Big)-f_1\Big(t-\frac{\theta}{2}\Big)\Big\}-\Big\{4f_3\Big(t-\frac{3\theta}{2}\Big)-5f_2\Big(t-\frac{3\theta}{2}\Big)+f_1\Big(t\ \ \frac{3\theta}{2}\Big)\Big\}$$

$$+\Big\{12f_4\Big(t-\frac{5\theta}{2}\Big)\ \ 20f_3\Big(t-\frac{5\theta}{2}\Big)+9f_2\Big(t-\frac{5\theta}{2}\Big)-f_1\Big(t-\frac{5\theta}{2}\Big)\Big\}-\ldots\Big]$$

$$+4k^2[\{f_3(t_1)\ \ f_2(t_1)\}\ \ \{6f_4(t_2)-8f_3(t_2)+2f_2(t_2)\}+\ldots\ \ ]$$

$$+8k^3\Big[\Big\{f_4\Big(t-\frac{3\theta}{2}\Big)-f_3\Big(t\ \ \frac{3\theta}{2}\Big)\Big\}-\Big\{8f_5\Big(t-\frac{5\theta}{2}\Big)-11f_4\Big(t-\frac{5\theta}{2}\Big)$$

$$+3f_3\Big(t-\frac{5\theta}{2}\Big)\Big\}+\ldots\ \ \Big]+16k^4[\{f_5(t_2)-f_4(t_2)\}-\ldots]$$

Equation (15) is the general expression for displacement of the struck point during impact. Any function in the above expression will appear only from the time obtained by equating the corresponding argument to zero.

It is clear from the above equation that at intervals of a period (i.e. $2a/c$) two types of reflected wave generate in succession— two waves similar to those in case of a flexible string, and two more, 'now stiffness controlled' proceeding towards the struck point, where new waves (represented by f_2, f_3 etc) are continuously being generated. It is obvious that the functions representing the waves generated during impact only will exist in the value of y_a.

The value of the functions occurring in equation (15) is the same as those obtained by Ghosh (1953).

If the tension be infinitely large or if the stiffness of the string be zero, we get,

$$f_1(t) = \frac{v_0}{\beta}(1-e^{-\beta t}) = \frac{v}{q}(1-e^{-qt}) \qquad \ldots \quad (16.1)$$

$$f_2(t) = \frac{v_0}{\beta}\cdot \beta t e^{-\beta t} = \frac{v_0}{q}\cdot qt.e^{-qt} \qquad \ldots \quad (17.2)$$

$$f_3(t) = \frac{v_0}{\beta}\left(\beta t - \frac{\beta^2 t^2}{2}\right)e^{-\beta t} = \frac{v_0}{q}\left(qt - \frac{q^2t^2}{2}\right)e^{-qt} \qquad \ldots \quad (18.3)$$

and so on.

Those values of $f_1(t)$, $f_2(t)$ $f_3(t)$ etc. as obtained in (16.1), (17.2) and (18.3) etc. are similar to those obtained by Ghosh. (loc. cit).

The first term of equation (15) is zero for negative values of time for positive values, i.e.,

$$0 < t < \theta,$$

$$y_a = f_1(t) + 2k\left[f_2\left(t-\frac{\theta}{2}\right) - f_1\left(t-\frac{\theta}{2}\right)\right] \qquad (18.1)$$

During, $\quad \theta < t < 2\theta.$

$$y_a = y_a(0 < t < \theta) - \{2f_2(t_1) - 2f_1(t_1)\}$$
$$-2k\left\{4f_3\left(t-\frac{3\theta}{2}\right) - 5f_2\left(t-\frac{3\theta}{2}\right) + f_1\left(t-\frac{3\theta}{2}\right)\right\}$$
$$+4k^2\{f_3(t_1) - f_2(t_1)\} + 8k^3\left\{f_4\left(t-\frac{3\theta}{2}\right) - f_3\left(t-\frac{3\theta}{2}\right)\right\} \qquad (18.2)$$

Similarly during. $\quad 2\theta < t < 3\theta$

$$y_a = y_a(\theta < t < 2\theta) + \{4f_3(t_2) - 6f_2(t_2) + 2f_1(t_2)\}$$
$$+2k\left\{12f_4\left(t-\frac{5\theta}{2}\right) - 20f_3\left(t-\frac{5\theta}{2}\right) + 9f_2\left(t-\frac{5\theta}{2}\right) - f_1\left(t-\frac{5\theta}{2}\right)\right\}$$
$$-4k^2\{6f_4(t_2) - 8f_3(t_2) + 2f_2(t_2)\}$$
$$-8k^3\left\{8f_5\left(t-\frac{5\theta}{2}\right) - 11f_4\left(t-\frac{5\theta}{2}\right) + 3f_3\left(t-\frac{5\theta}{2}\right)\right\}$$
$$+16k^4\{f_5(t_2) - f_4(t_2)\} \qquad \ldots \quad (18.3)$$

and so on.

The pressure exerted by the load during impact may now be easily obtained by using equations (5.0) and (18.1) (18.2) (18.3) successively for each epoch. By studying the pressure equations thus obtained and equating each to zero it wil be shown in a subsequent publication how to calculate the duration of impact at different intervals of time.

We now study the case when stiffness is the chief agent in the vibration.

Case II $$4EI\rho AD^2 > T^2$$

Here all the four roots of equation (2.1) are imaginary and the general solution of the equation can be written as,

$$y = A_1 \cosh \mu_1 x + A_2 \sinh \mu_1 x + A_3 \cos \mu_2 x + A_4 \sin \mu_2 x \qquad \dots \quad (20)$$

where,

$$\mu_1 = [\{(4EI\rho AD^2 - T^2)^{\frac{1}{2}} + T\}/2EI]^{\frac{1}{2}} \qquad \dots \quad (20.1)$$

$$\mu_2 = [\{(4EI\rho AD^2 - T^2)^{\frac{1}{2}} - T\}/2EI]^{\frac{1}{2}} \qquad \dots \quad (20.2)$$

and A_1, A_2, A_3, A_4 are constant to be determined from boundary conditions.

Equation (20) with the help of the boundary conditions (3.1), (3.2) and (3.3), becomes

$$y_1 = y_a \frac{\cosh \mu_1 x - \cos \mu_2 x}{\cosh \mu_1 a - \cos \mu_2 a}, \quad (0 < x < a) \qquad \dots \quad (21.0)$$

$$y_2 = y_a \frac{\cosh \mu_1 (l-x) - \cos \mu_2 (l-x)}{\cosh \mu_1 b - \cos \mu_2 b}, \quad (a < x < l) \qquad \dots \quad (21.1)$$

The hammer strikes at $x = a$ and the corresponding displacements are given by (21.0), (21.1). The pressure exerted by the load is given by

$$P = m \frac{d^2 y_a}{dt^2} \qquad \dots \quad (22.0)$$

and the subsequent equation of motion of the load is,

$$m \frac{d^2 y_a}{dt^2} = T\Delta \left(\frac{dy}{dx}\right)_{x=a} - EI\Delta \left(\frac{d^3 y}{dx^3}\right)_{x=a} \qquad \dots \quad (22.1)$$

Not substituting the values of $\Delta(dy/dx)_{x=a}$ and $\Delta(d^3y/dx^3)_{x=a}$ as obtained from (21.0) and (21.1) in equation (22.1) and imposing the boundary conditions we get,

$$mD^2 y_a = -Ty_a \left[\frac{\mu_1 \sinh \mu_1 b + \mu_2 \sin \mu_2 b}{\cosh \mu_1 b - \cos \mu_2 b} + \frac{\mu_1 \sinh \mu_1 a + \mu_2 \sin \mu_2 a}{\cosh \mu_1 a - \cos \mu_2 a} \right]$$

$$+ EIy_a \left[\frac{\mu_1^3 \sinh \mu_1 b - \mu_2^3 \sin \mu_2 b}{\cosh \mu_1 b - \cos \mu_2 b} + \frac{\mu_1^3 \sinh \mu_1 a - \mu_2^3 \sin \mu_2 a}{\cosh \mu_1 a - \cos \mu_2 a} \right] + DJ \qquad (22.2)$$

HAMMER STRIKES AT THE MIDDLE POINT OF THE STRING

In this case $a = b$, equation (22.2) now takes up the form,

$$mD^2y_a = -2Ty_a\left[\frac{\mu_1\sinh\mu_1 a + \mu_2\sin\mu_2 a}{\cosh\mu_1 a - \cos\mu_2 a}\right]$$

$$+ 2EIy_a\left[\frac{\mu_1^3\sinh\mu_1 a - \mu_2^3\sin\mu_2 a}{\cosh\mu_1 a - \cos\mu_2 a}\right] + DJ \qquad (22.3)$$

Replacing J by mv_0 we have from equation (22.3)

$$y_a = \frac{D}{F(D)}v_0 \qquad (23.0)$$

Where,

$$F(D) = D^2 + \frac{2}{m}\left[\frac{(T\mu_1 - EI\mu_1^3)\sinh\mu_1 a + (T\mu_2 + EI\mu_2^3)\sin\mu_2 a}{\cosh\mu_1 a - \cos\mu_2 a}\right] \qquad (23.1)$$

Now writing, $\qquad EI\mu_1^2 - T = EI\mu_2^2 \qquad (23.2)$

we have,

$$F(D) = D^2 + \frac{2EI}{m}\mu_1\mu_2\frac{\mu_1\sin\mu_2 a - \mu_2\sinh\mu_1 a}{\cosh\mu_1 a - \cos\mu_2 a} \qquad (23.3)$$

From equation (20.1) and (20.2), we have by expansion and retaining only upto linear power of EI the final of $F(D)$ to be,

$$F(D) = D^2\left[1 + \frac{M}{3m} - \frac{Ma^2T}{90mEI}\right] + \frac{Ma^4\rho A}{1890mEI}D^4 \qquad \ldots \quad (23.4)$$

where, $\qquad M = 2a\rho A$

Thus equation (23.0) can be written in the form with the help (23.4).

$$y_a = \frac{v_0}{\beta}\cdot\frac{1}{D(D^2+\alpha^2)} = \frac{v_0}{\beta\alpha^2}\left[t - \frac{1}{\alpha}\sin\alpha t\right] \qquad \ldots \quad (24.0)$$

where, $$\alpha = \left[\frac{1}{\beta}\left(1 + \frac{M}{3m} - \frac{Ma^2T}{90mEI}\right)^{\frac{1}{2}}\right] \qquad \ldots \quad (24.1)$$

$$\beta = \frac{Ma^4\rho A}{1890mEI} \qquad \ldots \quad (24.2)$$

Substituting the values of α and β in (24.0) we get,

$$y_a = \frac{v_0}{\left(1+\dfrac{M}{3m}-\dfrac{Ma^2T}{90mEI}\right)}\left[t-\frac{\sin\left\{\dfrac{1890mEI}{Ma^4\rho A}\left(1+\dfrac{M}{3m}-\dfrac{Ma^2T}{90mEI}\right)\right\}^{\frac{1}{2}}\cdot t}{\left\{\dfrac{1890mEI}{Ma^4\rho A}\left(1+\dfrac{M}{3m}-\dfrac{Ma^2T}{90mEI}\right)\right\}^{\frac{1}{2}}}\right] \quad \ldots \quad (24.3)$$

If now the stiffness of the string is very large compared to tension (24.3) becomes,

$$y_a = \frac{v_0}{\left(1+\dfrac{M}{3m}\right)}\cdot t \quad \ldots \quad (24.4$$

Equation (24.4) shows that when stiffness is very large the string behaves as a rigid rod, a conclusion similar to that derived by Ghosh (1938) in the case of pianoforte string of very short length.

Pressure exerted by the load during impact as obtained by equation (22.0) and (24.0), is given by,

$$P = mD^2y_a$$

$$= \frac{mv_0}{\beta\alpha}\sin\alpha t. \quad \ldots \quad (25.0)$$

Thus the pressure exerted by the load on the string at the beginning is zero and attains a maximum value $mv_0/\beta\alpha$ after a time $\pi/2\alpha$ and the pressure falls to zero at the end of time $t=\phi$ the duration of impact, given by

$$\phi = \frac{\pi}{\alpha} = \frac{\pi}{\left[\dfrac{1890mEI}{Ma^4\rho A}\left(1+\dfrac{M}{3m}\right)-\dfrac{21c^2}{a^2}\right]^{\frac{1}{2}}} \quad \ldots \quad (26)$$

ACKNOWLEDGMENT

Tke author expresses his thanks to Dr. S. K. Ghosh Reader of Physics, Jadavpur University, for guidance in preparing the paper.

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